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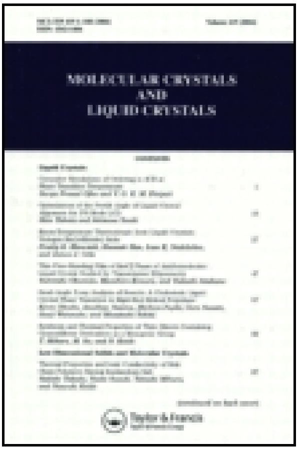

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“Magic Spiral” Submitted to a Torque: Topological Flows Driven by Ericksen Stresses in Sm C Films

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"Magic Spiral" Submitted to a Torque : Topological Flows Driven by Ericksen Stresses in Sm C Films

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We will show here that, in a two-dimensional liquid crystal system where disclinations are present, the elastic torque cannot balance alone an external torque ; a hydrodynamic flow must indeed occur so that the system can reach a stationary state. We suggest calling such flows **topological flows**. We will indicate how they are produced by the Ericksen forces.

1. REACTIVE COUPLING BETWEEN THE ORDER PARAMETER AND FLOWS VIA ERICKSEN STRESSES

The hydrodynamics of the liquid crystals deals not only with flows of matter but also with the dynamics of the order parameters resulting from broken symmetries such as rotational and/or translational symmetries [1]. Because of the Onsager relations binding the fluxes and the forces defined within the framework of the non-equilibrium thermodynamics, the temporal evolutions of the flow and of the order parameters are coupled. The resulting complex phenomenology is furthermore entangled by the presence of topological defects. In the nematics, the director field $\mathbf{n}(\mathbf{r},t)$ and the velocity field $\mathbf{v}(\mathbf{r},t)$ are reciprocally coupled by dissipative (viscous) and reactive (elastic) terms occurring in the equations of motion. The coupling **via the dissipative terms** was extensively studied : it is known that 1°- due to the viscous torque, the director field **n** can be oriented by the flow **v** and, conversely, 2°- due to the viscous stresses, the director field **n** has an influence on the flows (anisotropy of viscosity, hydrodynamic instabilities, back-flow effects). The existence of this coupling is obvious in this case since the viscous torque and the viscous stresses depend simultaneously on fields **n** and **v**.

In this paper, we want to focus our attention on the reciprocal coupling of **n** and **v via the reactive terms** ; these terms are the elastic torque in the equation of motion of the director and the elastic stresses known as the Ericksen stresses in the equation of motion of the fluid. In this case, the existence of a coupling between **n** and **v** is not obvious any more since both the elastic torque and the Ericksen stresses only depend explicitly on the director field **n**. A

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diagram explaining this kind of coupling is given fig. 1 : an external torque Γ distorts the director field **n**. This distortion generates Ericksen forces $\mathbf{f}^{Er}$ in the bulk which in turn induce a flow **v** in the system.

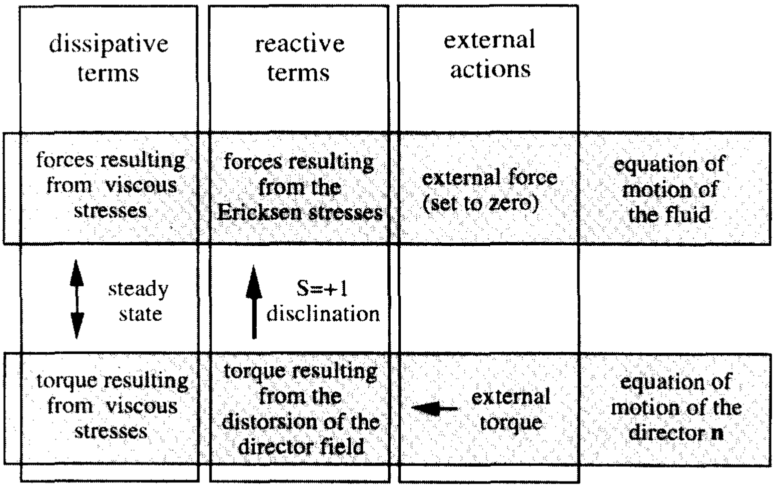

FIGURE 1: Scheme of dissipative and reactive couplings between flows and the order parameter

In order to give a convincing demonstration of the action of the Ericksen forces, one has to eliminate all the effects due to the dissipative terms. For this reason, all the transient processes in which the director field is not static and can generate "back-flows" must be excluded. The director field **n**(**r**,t) can be static (time-independent) either at equilibrium or in a stationary state. At equilibrium, the Ericksen forces can always be balanced by a suitable field of pressure so that no flow is produced [1]. For our demonstration we will therefore consider the second situation where a stationary state is induced by an external torque.

We will show here that in the presence of disclinations, the external torque cannot be balanced by the elastic torque alone and that the system can only reach a stationary state in which permanent flows appear. We propose to call such flows **topological flows**. We will point out that these flows are generated by the Ericksen forces.

The generic example of an unusual hydrodynamic behaviour in the presence of topological defects is provided by the free-standing smectic C films [2]. Because of the very small thickness of these films as compared with their lateral dimensions, the hydrodynamic flows are two-dimensional here. In a reference frame with the z-axis perpendicular to the plan of the film, one has:

$$v_x = v_x(x,y,t), \quad v_y = v_y(x,y,t) \quad \text{and} \quad v_z = 0 \tag{1}$$

These 2D flows are coupled with the 2D orientational order parameter $\mathbf{c}(x,y,t)$ (fig.2).

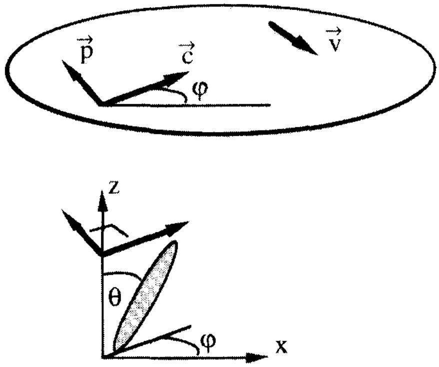

FIGURE 2 : Flows and order parameter in SmC films

In the approximation of a constant tilt angle θ, the amplitude $|\mathbf{c}| = \sin\theta$ of the two-dimensional order parameter remains unchanged so that the phase $\varphi(x,y,t)$, i.e. the azimutal angle, is sufficient to describe the orientational order of the system. Under the same approximation, the smectic C films are incompressible ; thus, in the absence of external sources, the two components of the velocity field are linked by the matter conservation law :

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0 \tag{2}$$

In conclusion, a smectic C film can be treated like a two-dimensional nematic. For this reason, we will adopt in this paper the notations used in the nematohydrodynamics.

It has been shown for the first time by Cladis et al. [2] that the phenomena resulting from the coupling of the divergence-less flow $\mathbf{v}(x,y,t)$ and the phase field $\varphi(x,y,t)$ depend on the presence of a $(+2\pi)$ disclination in the film. In these first experiments, a circular SmC film of radius R was pierced in its centre by a thin needle of radius $r_i < R$ (fig. 3.a). In the case, when the phase field at rest is defectless, for example uniform with $\varphi(r,\psi,0)=0$ (in the cylindrical coordinates (r,ψ,z)), the rotation of the needle by an angle $\Phi(t)$, results in a "winding" of the phase : at $r= r_i$ the phase rotates with the needle so that $\varphi(r_i,\psi,t)=\Phi(t)$.

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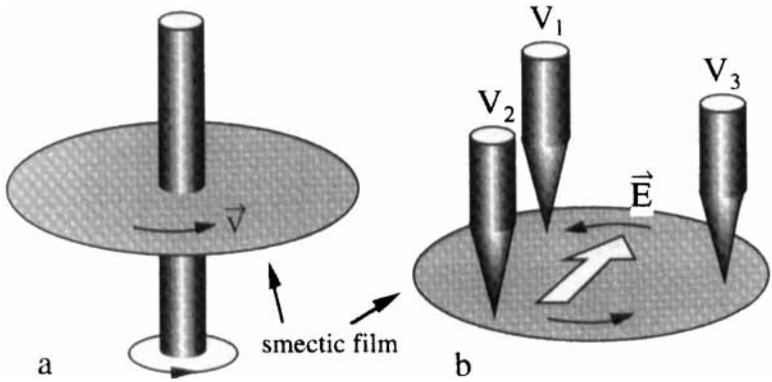

FIGURE 3 : Cladis' et al. experiments [2,3] : a- flows driven by a rotating needle, b- rotating electric field applied to the SmC film.

At the external edge of the film, the phase remains unchanged $\varphi(R,\psi,t)=0$. Between R and r_i it varies continuously as a function of r and t. When the needle is withdrawn from the film, the phase relaxes towards zero everywhere in the system.
The behaviour of the phase field is quite different when this one contains at rest a $(+2\pi)$ defect, for example $\varphi(r,\psi,0)=\psi$. In such a case, the rotation of the needle does not wind up the phase : this one tends towards a non-zero stationary value $\varphi=\psi+\Delta\varphi$ (where $0<\Delta\varphi<\pi/2$), except at the boundaries. One says that there is flow alignment in the SmC film .
Recently, Cladis et al. [3] replaced the rotating needle by a rotating electric field (fig.3b) and studied its influence on the behaviour of the phase field in the presence of a $(+2\pi)$ disclination. The angular velocity of the applied field was large enough so that the phase φ was subjected to a time-averaged electric torque. Without field, the equilibrium position of the disclination is located at the centre of the film. Cladis et al. have shown that, above a threshold value Γ_{cr} of the torque Γ, the disclination is orbiting around a target-like pattern formed in the middle part of the film. The phase, created at a constant rate $\partial\varphi/\partial t$ in the centre of the target is annihilated, at the same rate, by the orbiting disclination.
The aim of this paper is to extend the study of Cladis et al. to the case of a S=+1 disclination carrying an inclusion in its core. In the experimental section, we will show that for $\Gamma<\Gamma_{cr}$, the disclination, and thus the director field, are static. Nevertheless, steady flows (topological flows) are induced by the action of the

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rotating electric field. In the theoretical section, we will specify that these flows are produced by forces resulting from the Ericksen stresses.

II. MAGIC SPIRAL SUBMITTED TO A TORQUE

The starting point of our theoretical considerations is the so-called "magic spiral" paradox [1]. Transposed to the smectic C films, it can be formulated as follows : one considers a film drawn on a circular frame of radius R (see fig. 4). This film contains a small solid inclusion of a radius r_i suspended in its centre. The inclusion and the frame exert an anchoring action on the phase field. We suppose that the director **c** is orthogonal to these two boundaries.

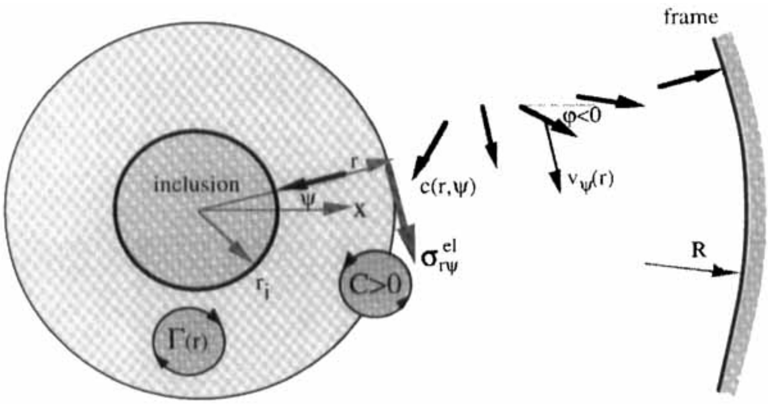

FIGURE 4 : The Magic Spiral submitted to a torque, definitions.

II.1. Equilibrium in the absence of the external torques

At equilibrium (in the absence of external torque), the phase field $\varphi(r,\psi)$ has to satisfy the Laplace equation

$$\Gamma = K\left(\frac{1}{r^2}\frac{\partial^2\varphi}{\partial\psi^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\varphi}{\partial r}\right)\right) = 0 \tag{3}$$

which expresses the vanishing of the elastic torque in the approximation of the isotropic elasticity. Solutions of (3), compatible with the boundary conditions, are :

$$\varphi = \varphi_{ns}(r) + S\psi \ ; \quad S=+1 \tag{4}$$

where

$$\varphi_{ns}(r) = (W\pi)\frac{\ln(r/R)}{\ln(r_i/R)}; \qquad W=0\pm1,\pm2,... \tag{5}$$

corresponds to the **non singular** component of the distortion. The integer W that we propose to call the **winding number** is a topological invariant of the phase field in the case of a strong anchoring (W=-1 on fig. 4).

Due to this distortion, there is an elastic torque acting on the edge of the inclusion :

$$C = K\frac{\partial\varphi}{\partial r} = K\frac{\partial\varphi_{ns}}{\partial r} \tag{6}$$

When $\partial\varphi/\partial r>0$, this torque is positive (anticlockwise), as shown on fig. 4.

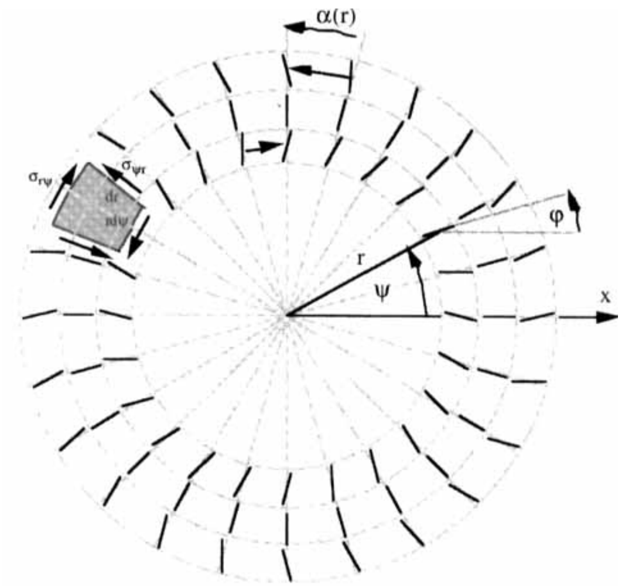

FIGURE 5: Genesis of the Ericksen stresses

If there were no other contribution to the torque exerted on the inclusion, then the elastic torque C would induce a perpetual solid-like rotation of the inclusion. Fortunately, in addition to this "direct" elastic torque, one has to consider the torque resulting from the action of the elastic stresses on the edge of the disk

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(see [11]). The elastic stress tensor, also known as the Ericksen stress tensor, expresses the response of the LC to a deformation of the director field $\varphi(\mathbf{r})$ which consists in a displacement $\mathbf{u}(\mathbf{r})$ of the molecules preserving their orientation :

$$\varphi'(\mathbf{r}+\mathbf{u}(\mathbf{r}))=\varphi(\mathbf{r}) \quad (7)$$

Its origin appears clearly from the example shown on fig. 5. When the molecules, drawn in bold type, are transported as indicated on the figure, the elastic deformation $\partial\varphi/\partial r$ is relaxed by the amount $-(\partial\varphi/\partial\psi)(\partial\alpha/\partial r)$. Let's emphasize that, in the absence of anchoring at the edges of the film, the elastic energy stored in the logarithmic phase field, $\varphi=\varphi_0 \ln(r/R)$, could be completely relaxed by the deformation $\alpha(r)=\varphi_0 \ln(r/R)$.

The Ericksen stress component $\sigma^{Er}_{r\psi}$ exerted by the LC on the edge of the inclusion is:

$$\sigma^{el}_{r\psi} = -K\frac{\partial\varphi}{r\partial\psi}\frac{\partial\varphi}{\partial r} = -K\frac{S}{r}\frac{\partial\varphi_{ns}}{\partial r} \quad (8)$$

This stress produces a negative (clockwise) torque which cancels the "direct" elastic torque calculated above (see eq.6) for S=1. Therefore, as mentioned in ref.[11], the perpetual motion paradox is avoided. The Ericksen stresses occur also in the bulk of the film and can produce a force per unit area. In order to calculate this force in the case of the magic spiral configuration, one has to take into account all stresses $\sigma^{Er}_{r\psi}$ and $\sigma^{Er}_{\psi r}$ acting on a surface element $rd\psi dr$. As a result, one gets :

$$f_\psi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\sigma_{r\psi}\right) + \frac{\sigma_{\psi r}}{r} \quad (9)$$

In the magic spiral configuration, the component $\sigma^{Er}_{\psi r}$ of the Ericksen stress tensor has the same form as $\sigma^{Er}_{r\psi}$ given in eq. (8) so that

$$f_\psi = -K\frac{S}{r}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\varphi}{\partial r}\right) \quad (10)$$

Let's note that, except for the factor -S/r, this expression of the Ericksen force is identical to the expression of the "direct" elastic torque $K\Delta\varphi$. For this reason, in the case of the magic spiral (S=+1) at equilibrium (in the absence of external torque), the elastic torque and the Ericksen force vanish simultaneously.

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II.2. Action of an external torque

Let us suppose now that the magic spiral configuration is subjected to an external torque per unit area $\Gamma(r) = \Gamma \neq 0$. If there were no flow, the steady state of the phase field could not be reached. This can be simply demonstrated by the following discussion. Let us assume that v=0 and $\partial\varphi/\partial t=0$; there is therefore no viscous torque and the external torque Γ acting on the director **c** is then balanced by the elastic torque

$$K\Delta\varphi=-\Gamma \tag{11}$$

Under the same conditions, the equation (10) tells us that the Ericksen force f^{Er}_{ψ} acting on the fluid is $S\Gamma/r \neq 0$ and must induce flows in the film. This is incompatible with the assumption v=0 ; thus this assumption is bad.
When flows exist, one must take into account the viscous torque acting on the director. In the simplest case when the viscosity coefficient γ_2 is set to zero, this torque results only from the difference

$$N = \left(\frac{\partial\varphi}{\partial t} + v_{\psi} \frac{\partial\varphi}{r\partial\psi} \right) - \frac{1}{2r}\frac{\partial(rv_{\psi})}{\partial r} \tag{12}$$

between the angular velocities of the director and of the fluid so that the equation of motion of the director writes :

$$-\gamma_1 N + K\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\varphi}{\partial r}\right) + \Gamma = 0 \tag{13a}$$

In the same approximation, the equation of motion of the fluid, written as the balance of torques rf_{ψ} due to the body forces f_{ψ}, has the form :

$$\gamma_1 N + r\eta\left(\frac{\partial^2 v_{\psi}}{\partial r^2} + \frac{1}{r}\frac{\partial v_{\psi}}{\partial r} - \frac{v_{\psi}}{r^2} \right) - KS\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\varphi}{\partial r}\right) = 0 \tag{13b}$$

Besides the torque due to the Ericksen force (last term), this equation contains the two contributions due to the viscous stresses. The first term corresponds to the antisymmetric part of the viscous stress tensor and represents the torque per unit area + γ_1N which is the counterpart of the viscous torque - γ_1N felt by the molecules and present in the equation (13a). The second term is due to the symmetrical part of the viscous stress tensor.

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The equations (13a) and (13b) contain the two unknown functions $v_\psi(r)$ and $\varphi(r)$. First of all let us find $v_\psi(r)$. We calculate $K\Delta\varphi$ from 13a and substitute it in 13b :

$$\gamma_1 N + r\eta\left(\frac{\partial^2 v_\psi}{\partial r^2} + \frac{1}{r}\frac{\partial v_\psi}{\partial r} - \frac{v_\psi}{r^2}\right) - S(\gamma_1 N - \Gamma) = 0 \qquad (14)$$

In the case of the magic spiral, S=1, so that the $\gamma_1 N$ terms vanish ; we are then left with

$$r\eta\left(\frac{\partial^2 v_\psi}{\partial r^2} + \frac{1}{r}\frac{\partial v_\psi}{\partial r} - \frac{v_\psi}{r^2}\right) = -\Gamma \qquad (15)$$

This means that the shear viscosity is opposed to the torque Γ. In the approximation where the radius of the inclusion is very small (r_i <<R), we can now calculate the flow field :

$$v_\psi = -\frac{\Gamma R}{2\eta}\tilde{r}\ln(\tilde{r}) \quad with \quad \tilde{r} = \frac{r}{R} \qquad (16)$$

When Γ<0 (clockwise from its definition - see fig. 4), the velocities v_ψ are negative (ln(r/R)<0) so that the circulation in this flow pattern should have the same sense as the external torque (see fig.4). The advection of the phase due to such a flow results in the rotation of the molecules in the sense of the external torque, as expected.

Knowing the flow pattern, we can now calculate the distortion of the phase field from the equation (13a) :

$$-\gamma_1\left[(S-1)\frac{v_\psi}{r} - \frac{1}{2}\left(\frac{\partial v_\psi}{\partial r} - \frac{v_\psi}{r}\right)\right] + K\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\varphi}{\partial r}\right) + \Gamma = 0 \qquad (17)$$

With S=1, and v_ψ given by eq.(16), one obtains:

$$K\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\varphi}{\partial r}\right) = -\Gamma\left(1 - \frac{\gamma_1}{4\eta}\right) = -\tilde{\Gamma} \qquad (18)$$

This equation tells us that the fraction ($\gamma_1/4\eta$) of the external torque Γ is dissipated in the persistent flow and only the remaining fraction of the torque Γ distorts the logarithmic static configuration of the magic spiral.
If the inclusion S=+1 is located at the film centre, the distortion $\varphi(r)$ has the form:

$$\varphi = \varphi_{ns}(r) + S\psi \; ; \quad S=+1 \tag{19}$$

where the non singular component $\varphi_{ns}(r)$ satisfying the equation (18) must have the following form

$$\varphi_{ns}(r) = (W\pi)\left[\alpha \frac{\ln(r/R)}{\ln(r_i/R)} + \beta \frac{R^2 - r^2}{R^2 - r_i^2}\right]; \tag{20}$$

with

$$W=0, \pm 1, \pm 2,. \text{ and } \alpha+\beta=1$$

When the torque Γ is zero, α=1 and the distortion is purely logarithmic. In the presence of the torque Γ, some fraction β of the total phase winding becomes parabolic. From equation (18), one gets :

$$\beta W\pi = \frac{\Gamma}{4K}\left(R^2 - r_i^2\right) \tag{21}$$

For W<0 (see fig. 4 where W=-1) and in the presence of the effective torque $\tilde{\Gamma}<0$, β is positive so that the distortion due to the phase winding $W\pi$ partially loses its logarithmic, torqueless form ; it gradually evolves, with the effective torque value $\tilde{\Gamma}$, into a "parabolic" distortion. For $\tilde{\Gamma}=\tilde{\Gamma}_{cr}$ such that β =1 and α=0, the distortion loses its logarithmic character completely. Finally, for $\tilde{\Gamma}>\tilde{\Gamma}_{cr}$, α becomes negative which means that the logarithmic contribution to the distortion changes its sign in order to satisfy the conservation of the winding number. In the accompanying paper [8], we show that the inversion of the sign of the distortion can be avoided by moving the inclusion away from the film centre.

In conclusion : **when the magic spiral is subjected to an external torque, its nonsingular phase field loses its logarithmic character ; some fraction of the phase winding is transformed by the external torque into a parabolic distortion. This transformation is accompanied by flows. These flows never stop - they persist in the steady state when the phase field is static.**

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II.3. Relaxation of the magic spiral

Let us suppose now that the external torque Γ is removed. The distortion that it created must now relax which means that the molecules will rotate backwards. This backward rotation is led by the elastic torque (its initial value is $-\tilde{\Gamma}$) whose sense is opposite to the sense of Γ. The angular velocity of the molecules $\partial\varphi/\partial t$ creates viscous torques and viscous stresses which could generate back-flows [4]. Does this relaxation have to be accompanied by back-flows ? The answer is NOT because the contributions of the stresses due to the γ_1 term in the equation of motion of the fluid (13b) vanishes in the presence of the S=+1 disclination. The equation (13b) tells us that **no back-flow can occur during the relaxation of the magic spiral**. This theoretical conclusion is confirmed by experiments (section III). The relaxation process is therefore described by the equation (13a) in which v_ψ is set to zero:

$$-\gamma_1 \frac{\partial \varphi}{\partial t} + K \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial \varphi}{\partial r} \right) = 0 \tag{22}$$

At the beginning of the relaxation, the angular velocity $\partial\varphi/\partial t$ is the same everywhere. Later on, the relaxing phase field takes the form of a Bessel function and the relaxation becomes exponential.

III. EXPERIMENTAL

III.1. Set-up

In the experiments we used the room-temperature SmC* mixture SCE4 from BDH. Smectic films of dimensions 3mm×3mm were drawn on a rectangular frame with movable edges (fig.6b) which is described in details in ref.[6]. The typical thickness of these films was about 20 molecular layers (600Å). The frame was laid on the stage of a reflecting polarizing microscope.

The films were observed, through microscope, with a TV camera connected to an AV computer which allows to record either selected pictures or series of pictures taken at a regular time interval.

The rotating electric field was produced by four needle-shaped electrodes, placed below the film and forming a square as indicated on the figures.6.c and d. The electrodes were centred on the field of the microscope and their distance with respect to the film was controlled by a translation stage. The electrodes from the opposite corners of the square (0.3×0.3 mm) form two pairs which are supplied by AC voltages in phase quadrature: $V\sin(\omega t)$ for one pair and $V\cos(\omega t)$ for the other one. The frequency ω is about 1 kHz. This set of electrodes generates an electric field of a quite complex geometry. For the purpose of the present experiments, we are only interested in the component $E_{//}$

of the electric field tangent to the film. For symmetry reason, the field must be parallel to the film at the centre of the square but has a component E_z normal to the film elsewhere.

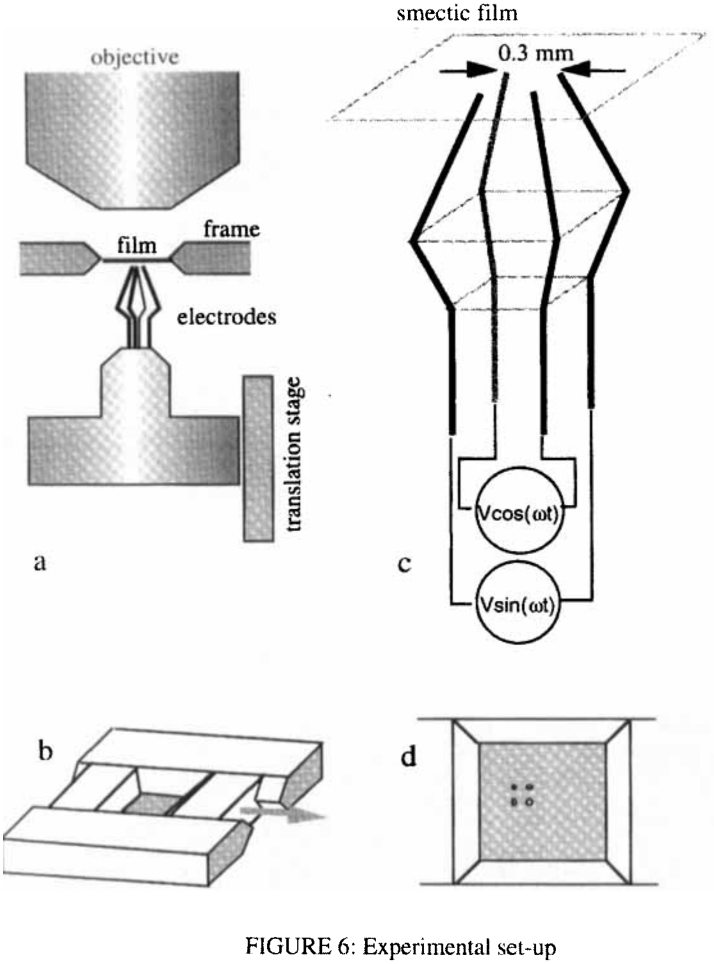

FIGURE 6: Experimental set-up

One has also to consider the polarization of the field. It is circular in the centre of the square but elliptic everywhere else. In a very rough approximation, one can suppose that the torque exerted by the rotating field is confined to a small

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region in the centre of the square. This localization of the torque has an advantage : as we will see below, it is difficult to control the position of the disclinations in the film ; however, since the frame supporting the film can be translated (using the microscope stage) with respect to the electrodes, the isolated disclinations (or the groups of disclinations) can always be positioned in the small area where the electric torque is localized. In this way one can study the individual or collective response of the disclinations to the external torque.

IV.2. Preparation of films containing disclinations

It is well known that the steps corresponding to the changes in the film thickness exert an anchoring action on the vector field **c** [7]. This property can be used for the preparation of smectic C films containing only one $(+2\pi)$ disclination.

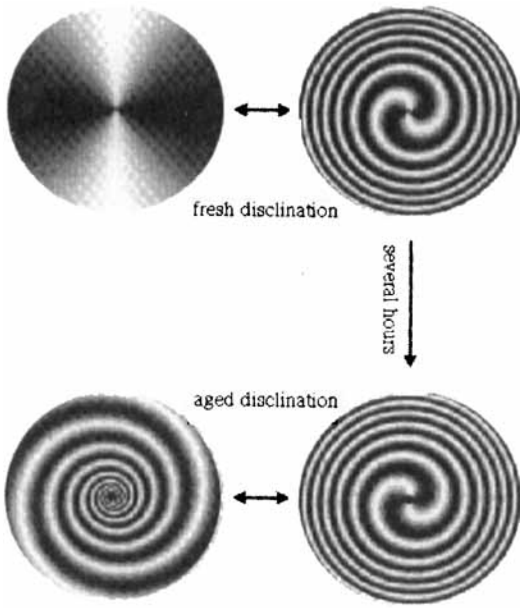

Fig.7: Preparation of an aged disclination with a blocked winding: 1°- a fresh disclination is winded up with the electric torque, 2°- after several hours, dust particles are accumulated in the disclination's core - the winding is blocked; when the torque is suppressed, the disclination keeps its winding number, the distortion field relaxes, its logarithmic configuration.

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One has to proceed as follows : during the initial stage of the smectic film realization, i.e. when one starts to open the rectangular frame, a small (≈100 μm in diameter) disk-shaped thin nucleus of smectic film (surrounded by a thick meniscus) is created. The topology of the vector field **c** in this nucleus can be easily detected by direct observation in the reflecting polarizing microscope. In many cases, the nucleus of the film contains only one (+2π) disclination. When it is true, one can draw a film of much larger dimensions by moving very slowly and smoothly the mobile edge of the frame. During this process, flows may occur in the film ; this makes the disclination drift but it does not matter since, due to the mobility of the frame supporting the film, the (+2π) disclination can always be positioned above the electrodes. We call the disclinations just created in this way "fresh" disclinations in opposition to the "aged" disclinations that have been staying in the film for several hours. On such a long time scale, the disclination traps the dust particles floating in the film and one can then observe a progressive growth of the disclination's core. This accumulation of dusts in the core of the disclination acts as the inclusion of radius r_i that we considered in the theoretical section : it exerts an anchoring action on the vector field **c**. The phase $\phi(r_i,\psi)=\psi+W\pi$ is blocked and cannot be altered by the action of the electric torque. In the fresh disclinations on the contrary, the SmA-like core structure does not impose any constraint on the phase.

However, in the (+2π) disclinations, there are energy barriers (due to the anisotropy of the elastic constants) to overcome in order to wind up the phase. It is nevertheless possible when the applied electric torque is large enough. The phase winding process is then quite visible in the polarizing microscope : the extinction brushes of the disclination are winded into a spiral as shown on the scheme of fig.7. This winding process is partially reversible in the fresh disclinations : when the torque is suppressed, one observes the unwinding of the phase during which the existence of energy barriers is obvious. However, when a disclination is maintained in this winded state during several hours, the accumulation of dust particles in the core blocks irreversibly the phase. An example of such a winded and aged disclination is shown on the photograph presented fig.8.

III.3. Topological flows, results

The topological flows induced by the electric torque were detected by looking at the motion of small dust particles floating in the film. Their positions were determined from photographs taken with the TV camera and recorded on the computer. A serie of pictures taken at a regular time interval were superimposed by using an image processing software. A typical example of such a composite picture is shown on fig. 8. On this image, the various positions of a dust particle are indicated by white dots. One can notice that the distortion field does not change throughout the experiment which means that the system was in a steady state. The dust particles evolve, in this stationary distortion field, on circular orbits around the disclination.

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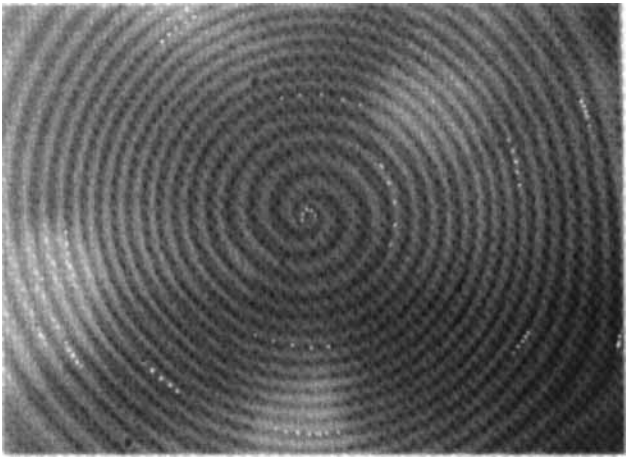

FIGURE 8: Topological flows around a (+2π) disclination. This picture is a superposition of five images taken at time intervals of 20 sec. The positions of dust particles are indicated by white dots. The flow is circular and its velocity depends on the distance to the disclination core.

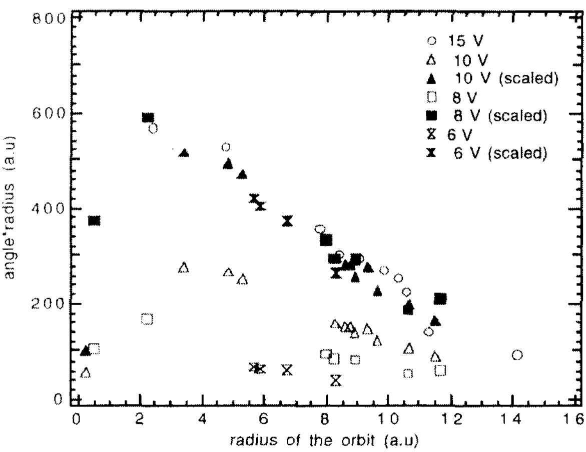

FIGURE 9 : Flow velocity (radius times angle of rotation) versus the distance from the disclination centre (radius of the orbit).

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The sense of circulation is similar to the sense of rotation of the electric field. This is in agreement with the theoretical scheme on fig.4. The velocities of the particles are constant. Typically, for a radius D≈0.15 mm of the orbit, the period of the orbit is T≈500 sec, and the velocity v≈1.9 µm/sec.
On the composite picture of fig.8, several dust particles evolve on circular orbits of different radii. Their velocities have been plotted versus the radius of the orbit on fig.9. In this case, the voltage applied to the electrodes was 10 V. On the same plot we have also represented other data obtained for the following voltages : U=6, 8 and 15 V. All these data have been rescaled with respect to those corresponding to 15V ; the velocities have been multiplied by the factor factor $(U/15\ V)^2$. The rescaled data (filled markers) are aligned on the same curve which demonstrates that the torque exerted on the molecules by the rotating field varies as E^2. Thus, as expected, the averaged torque due to the spontaneous polarization of the SmC* phase is zero.

III.4. Relaxation of the magic spiral

After the suppression of the external torque, the non singular part of the phase field relaxes towards its logarithmic configuration. The theory developed in the section II predicts that, during this relaxation, the circular flow $v_\psi(r)$ should stop. This theoretical prediction is experimentally confirmed. In the serie of three photographs shown on fig. 10, the dust particle indicated by arrows has a radial trajectory. It does not circulate around the disclination but only follows the collapsing extinction brushes.

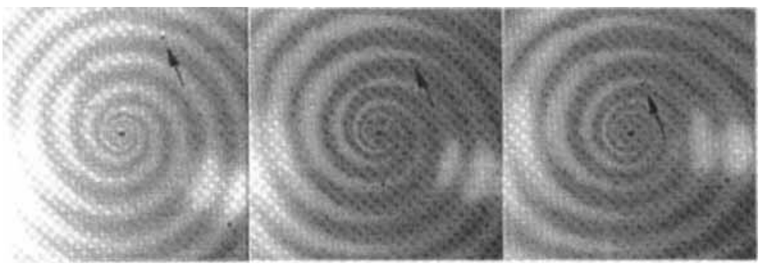

FIGURE 10 : Relaxation of the magic spiral after the suppression of the external torque. The dust particle indicated by the arrow does not circulate around the disclination : v_ψ=0. In its radial motion, the particle follows collapsing extinction brushes.

V. CONCLUSIONS

In this paper, we have shown, both experimentally and theoretically that an external torque, applied to the orientational order parameter **c** of the SmC films, cannot be balanced by a static deformation of the director field **c** when a (+2π) disclination is present in the film. Instead of an equilibrium state, the system can only reach a steady state in which permanent flows occur.

From a theoretical point of view, this conclusion follows directly from the expression of the two coupled equations (13a) and (13b). The motion equation of the fluid (13b) expresses the balance of the torques produced by the body forces acting on a fluid particle : it contains the term which corresponds to the elastic Ericksen force. This one depends explicitly on the topology of the director field ; it is proportional to the strength S of the disclination. Except for this factor -S, this term is identical to the elastic torque occuring in the motion equation of the director (13a). Consequently, when the director field, due to the external torque, leaves its equilibrium state, not only an elastic torque acting on the director but also body forces acting on the fluid particles are induced. It is obvious, starting from the equation (13b), that these Ericksen forces can only be balanced by the viscous forces. Therefore, the system can only reach a steady state in which flows must persist.

From the experimental point of view, the evidence for these **topological flows** is obtained by observing the motion of dust particles suspended in the SmC film ; in the stationary state, those are orbiting around the disclination with a velocity which depends on the radius of the orbit.

Another quite surprising consequence of the equations of motion (13a) and (13b) is that, after the suppression of the external torque, the system relaxes towards the equilibrium state without flow. Once again, this behaviour is due to the presence of the disclination in the director field. The equations of motion (13a) and (13b) can also be used to predict patterns of flows and distortions in the case when the director field contains a S=-1 disclination. In such a situation, it can be shown that topological flows are also induced but in a sense opposite to the external torque one : if the torque is clockwise, the circulation of the fluid should be now anticlockwise. The behaviour of S=-1 disclinations subjected to an external torque is considered in the accompanying paper [8].

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